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# Technical Notes

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## Analysis of Rocket Plume Base Heating by Using Backward Monte-Carlo Method

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### I. Introduction

WITH the requirement for payload capability of rockets, some advanced solid rocket motors have been developed with higher performance propellants. This increased performance, due mainly to increased aluminum loading, will increase the plume temperature, and hence enlarge the particle scatter effect. This means that the plume radiative emission and the base heating will increase. Therefore, the radiative base heating of rockets need to be checked further in detail.

Up to now, many methods for making practical calculations of radiative heating of rocket bases have been developed. Stockham and Love[1] employed the Monte Carlo method to investigate the thermal radiation from a cylindrical cloud of absorbing, emitting, and anisotropically scattering particles. The results show that anisotropic scattering and searchlight emission play an important role in rocket-plume base heating. Nelson[2] adopted the backward Monte Carlo method to predict radiative heating from conical isothermal uniform-temperature gray plumes with transparent boundaries and examined the effect of plume cone angle and searchlight emission. Baek and Kim[3] used the finite-volume method to analyze the radiative base heating of a rocket plume with uniform-temperature field and transparent boundary, due to searchlight emission and plume emission by changing various parameters. The results show that the base plane is predominantly heated by the plume emission, rather than the searchlight emission.

Recently, Liu[4] developed and validated a backward Monte Carlo method based on a radiation distribution factor for incident collimated irradiation, point, or line sources in a multidimensional enclosure with an absorbing, emitting, and scattering medium. This method has advantages over the backward Monte Carlo method used by Modest,[5] in the case where the radiative properties of the medium and boundary do not depend on temperature, such as radiative heating. The objective of this note is to use this method for the study of rocket-plume base heating in view of the anisotropic scattering and nonuniform temperature field in the plume.

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### II. Formulation

As shown in Fig. 1, the source of base heating is divided into two parts, namely searchlight emission and plume emission. Searchlight emission represents the radiant energy being emitted from the rocket nozzle and combustion chamber, and then being scattered by the particles in the plume, finally arriving at the base. This is an important aspect of plume radiation and it may contribute significantly to plume base heating. Plume emission, just as its name implies, is the radiant energy due to plume radiation.

Backward Monte Carlo methods state a termination, for example, a detector, and trace the photon bundle in the reverse direction, determining how much radiation power emitted along the path is incident on the termination site. For the calculation of base heating of a rocket due to radiation, the schematic of backward Monte Carlo ray tracing is shown in Fig. 1. The radiation flux from a point $x_p$ on the plume to a point $x_0$ on the base is defined as

$$q_0 = \int_{\Delta\Omega_l} I_{0l} \cos(\boldsymbol{n}_0 - \boldsymbol{s}_l)\, \mathrm{d}\Omega \tag{1}$$

where $\cos(\boldsymbol{n}_0 - \boldsymbol{s}_l)$ is the cosine value of the angle between the vector $\boldsymbol{s}_l$ along the line connecting point $x_p$ and $x_0$ and the outward normal vector $\boldsymbol{n}_0$ at $x_0$ on the rocket base plane, and $\Delta\Omega_l$ is the solid angle of the plume as viewed from the point $x_0$. According to the principle of the backward Monte Carlo method based on the radiation distribution factor,[4] radiative heat flux $q_0$ can be written as

$$q_0 = \sum_l \left[ \sum_i \sum_k \mathrm{RD}_{0ik} I_b(T_i) + \sum_j \sum_k \mathrm{RD}_{0jk} I_b(T_j) \right] \varepsilon_0 \cos(\boldsymbol{n}_0 - \boldsymbol{s}_l) \Delta\Omega_l \tag{2}$$

where $\mathrm{RD}_{0ik}$ and $\mathrm{RD}_{0jk}$ are the radiation distribution factors defined according to Ref. 4.

Because the base plane is assumed to be cold and black, the emissivity of the base plane, $\varepsilon_0$, is equal to 1. The solid angle $\Delta\Omega_l$ is computed by Monte Carlo theory from

$$\Delta\Omega_l = N_{\mathrm{abs}}/N_{\mathrm{emi}} \times 4\pi \tag{3}$$

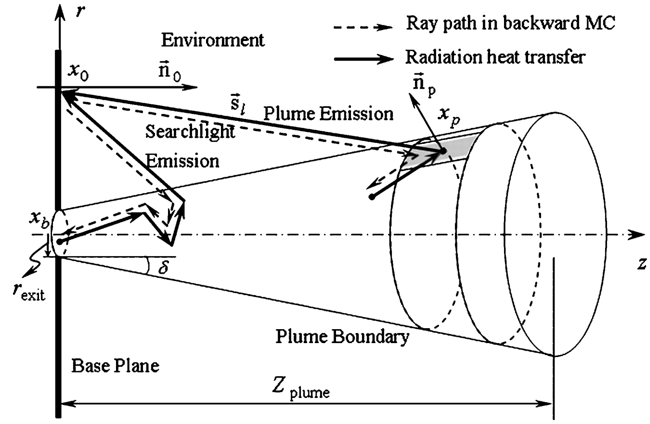


**Fig. 1 Schematic of rocket plume base heating.**

where $N_{abs}$ is the number of rays arriving at a certain surface of the plume and $N_{emi}$ is the number of rays emitted from the point $x_0$. The direction of the ray sent out from a point $x_0$ is determined by the polar angle $\theta_{x0}$ with values between 0 and $\pi$ and the azimuthal angle $\varphi_{x0}$ with values between 0 and $2\pi$. These angles are determined by picking two random numbers with values between 0 and 1.

The nondimensional radiative heat flux $\Theta_{flux}$ at the point $x_0$ of the base plane is defined as

$$\Theta_{flux} = q_0 \big/ \sigma T_{ref}^4 \tag{4}$$

where $T_{ref}$ is the reference temperature, namely the maximum temperature in the plume.

## III. Results and Discussion

A computer code based on the preceding calculation procedure was written. A number of ray-sampling studies were also performed for the physical model to ensure that the essential physics is independent of ray-sampling number. For the following numerical study, the number of rays emitted from a point in the base is $10^5$.

To examine the correctness of the code, the same case as given in Ref. 3 is calculated. The results are shown in Fig. 2 and compared with these obtained by Baek and Kim[3] using the finite volume method and Nelson[2] using the backward Monte Carlo method. From Fig. 2, it can be seen that the results are in very good agreement with the data in the references.

The parametric study will be presented in the next section. The parameters chosen include the scattering albedo $\omega_0$, scattering phase function $\Phi$, optical radius $\tau_0$, and transmittance of plume boundary $\tau_p$. For all of the cases discussed, the radius of the nozzle is $r_{exit} = 1.0$ cm, the plume length is $Z_{plume} = 50 r_{exit}$, and the plume cone angle is $\delta = 0$ deg. The temperature field in the plume is set as

$$T(z, r) = -(1400/17)(z - Z_{plume})(0.5 - 3r^2 + 2r^3) + 300\ (K) \tag{5}$$

#### Effect of Scattering Phase Function

In this study, forward and backward scattering phase functions are considered in addition to isotropic scattering, such as $\Phi = 1 + \cos\theta$, 1, and $1 - \cos\theta$. In Fig. 3, their effects on the radiative base heating are shown for the case of $\tau_0 = 0.5$, $\tau_p = 1.0$, and $\omega_0 = 0.9$. As seen from the figure, the forward scattering enhances the radiative energy transfer from the plume toward the base plane due to the effect of plume emission and the backward scattering causes an increase in the radiative heat flux on the base plane due to the effect of searchlight emission. The effect of isotropic scattering on the base heating is in the middle, but the scattering phase functions play only a minor role for the case of plume emission.

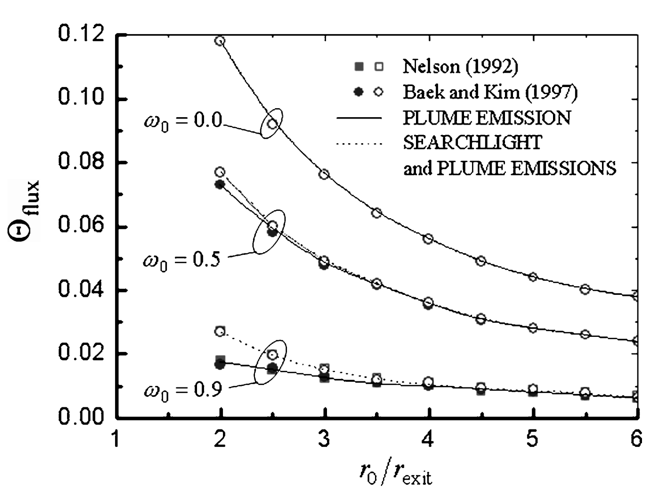


**Fig. 2 Effect of scattering albedo on base heating due to searchlight emission and/or plume emission in the case of $\Phi = 1$, $\tau_0 = 0.5$, and $\sigma = 0.450$ cm$^{-1}$. The temperature exhaust plume is the same as the nozzle exit temperature.**

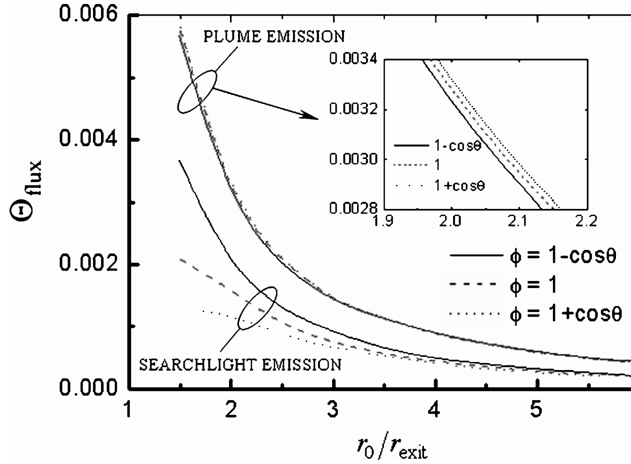


**Fig. 3 Effect of scattering phase function on base heating due to searchlight emission or plume emission in the case of $\omega_0 = 0.90$, $\tau_0 = 0.5$, and $\tau_p = 1.0$.**

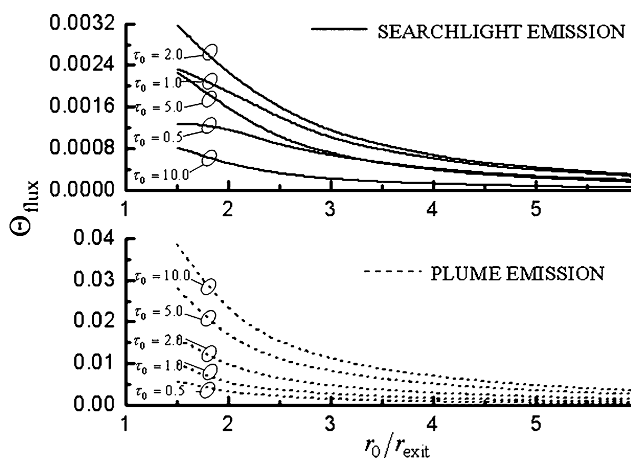


**Fig. 4 Effect of optical radius on the base heating due to searchlight or plume emission in the case of $\Phi = 1$, $\omega_0 = 0.90$, and $\tau_p = 1.0$.**

#### Effect of Optical Radius

The effect of optical radius on the base heating due to plume emission or searchlight emission for the case of $\Phi = 1$, $\tau_p = 1.0$, and $\omega_0 = 0.9$ is plotted in Fig. 4. Because the optical radius represents a ratio of the characteristic length of the system to the mean penetration length of the radiation, with increased optical radius, more radiation can be absorbed and emitted by the plume. As shown in Fig. 4, the radiative base heating due to plume emission at the base plane increases with the optical radius. However, the radiative base heating by searchlight emission initially increases and then decreases as the optical radius increases from 0.5 to 10.0.

#### Effect of Transmittance of Plume Boundary

The effect on boundary conditions due to the transmittance of the plume boundary is plotted in Fig. 5 for the case of $\Phi = 1$. Four different transmittances have been chosen. With decreased transmittance, the nondimensional radiative heat flux at the base plane is reduced. The transmittance of the plume boundary denotes a ratio of the bundles penetrating the boundary to the bundles arriving at the boundary; with decreased transmittance more radiation can be absorbed by the media. Therefore, more radiative energy can be reflected by the boundary and transferred to the plume.

Furthermore, as shown in Figs. 2 and 3, the nondimensional radiative heat flux at the base plane in view of the nonuniform temperature field is about two to three times less than the corresponding heat flux in view of the exhaust plume and the nozzle exit having the same

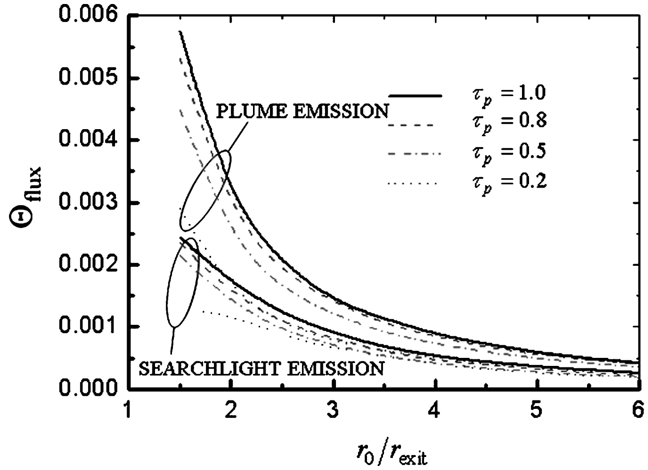


**Fig. 5 Effect of transmittance of plume boundary on the base heating due to searchlight emission or plume emission in the case of $\Phi = 1$, $\omega_0 = 0.90$, and $\tau_0 = 0.5$.**

temperature. The reason for these conclusions is that the temperature of the tail of the plume is lower than the temperature near the nozzle exit. Moreover, the assumption of an isothermal plume may overpredict the base heating flux by a factor of 2 or more.

## IV. Conclusions

A backward Monte Carlo code based on the radiation distribution factor has been applied to predict radiative base heating due to rocket plumes. For the case of uniform temperature in the plume with a transparent boundary, the calculation results show very good agreement with these obtained by Baek and Kim applying the finite-volume method and Nelson applying the reverse Monte Carlo method. Furthermore, predictions for an absorbing, emitting, and scattering plume with nonuniform temperature field in view of the effects of various parameters are presented. The base heating flux due to the rocket isothermal plume will be two to three times higher than the corresponding value due to the nonuniform temperature in the plume. As the plume albedo increases from zero toward one, the nondimensional heat flux at the base plane decreases. For the case of plume emission, while the forward scattering enhances the base heating, the backward scattering reduces it; for the case of searchlight emission, the forward scattering reduces the base heating, but the backward scattering enhances it. The radiative heat flux induced by plume emission increases as the optical radius increases from 0.5 to 10.0, but the searchlight emission initially increases and then decreases. As the transmittance of plume boundary decreases, the radiative heating reduces.

## Acknowledgment

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## References

[1]Stockham, L. W., and Love, T. J., “Radiative Heat Transfer from a Cylindrical Cloud of Particles,” *AIAA Journal*, Vol. 6, No. 10, 1968, pp. 1935–1940.

[2]Nelson, H. F., “Backward Monte Carlo Modeling for Rocket Plume Base Heating,” *Journal of Thermophysics and Heat Transfer*, Vol. 6, No. 3, 1992, pp. 556–558.

[3]Baek, S. W., and Kim, M. Y., “Analysis of Radiative Heating of a Rocket Plume Base with the Finite-Volume Method,” *International Journal of Heat and Mass Transfer*, Vol. 40, No. 7, 1997, pp. 1501–1508.

[4]Liu, L. H., “Backward Monte Carlo Method Based on Radiation Distribution Factor,” *Journal of Thermophysics and Heat Transfer*, Vol. 18, No. 1, 2004, pp. 151–153.

[5]Modest, M. F., “Backward Monte Carlo Simulation in Radiative Heat Transfer,” *Journal of Heat Transfer*, Vol. 125, No. 1, 2003, pp. 57–62.